

B.Sc semester-5 Inorganic chemistry  
US05CCHE22 Unit-1 Symmetry Lecture-11

By

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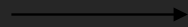
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## • ***Properties of point-groups*** :

- The point groups are studied with the help of the general mathematical technique called group theory. Let  $G$  be some group with  $g$  elements. Suppose, some set of elements, say  $H_1, H_2, \dots, H_h$  which are present in  $G$  are found to obey four rules of the point group. then this sets of elements  $H_1, H_2, \dots, H_h$  form a subgroup  $H$ . The number of elements in a group or subgroup is called order of the group. now take an element  $F$ , of the group  $G$ , which is absent in the subgroup  $H$  and form the products  $FH_1, FH_2, \dots, FH_h$ . They must all be in the group  $G$  but not in the subgroup  $H$ . If  $FH_i = FH_j$  were one of the members of  $H$  then  $F = H_j H_i^{-1}$  should also be in the subgroup  $H$ . This is contrary to our assumption concerning the selection of  $F$ . We have thus found  $2h$  members of the group  $G$ . If  $2h < g$ , it will be possible to find another element  $D$  not present among  $H_1, H_2, \dots, H_h$  nor among  $FH_1, FH_2, \dots, FH_h$ . Repeating the same arguments we obtain  $h$  new elements  $DH_1, DH_2, \dots, DH_h$ .

- Since the group  $G$  is of finite order, i.e,  $g$ , this procedure will end until



- $kh = g$
- where  $k$  is an integer. Thus, a group of six elements, for example, can only have subgroups of order either one or two or three.
- Que : A group of six elements can have subgroups of order either one(1) or two(2) or three(3) only.Explain.
- Ans : We know that  $k \times h = g$ ,  $k$  is an integer,  $g$  is the total number of member in the group and  $h$  is number of members in the subgroup (order),
- *This* value of six can be obtained in three ways  $6 \times 1 = 6, 3 \times 2 = 6,$  and  $2 \times 3 = 6$  i.e by putting  $h = 1, 2$  or  $3$ .
- *Hence*, A group of six elements can have subgroups of order either one(1) or two(2) or three(3) only.

## • **Conjugate Elements :**

- Two elements says, A and B, of a group are said to be conjugate if  $A = X^{-1} B X$  .....(i)
- where X is a another element of the same group. such transformation is known as similarity transformation.
- If we multiply A on its left side by X and on its right by  $X^{-1}$  and do the same on the right hand side of eq.(i), we have
  - $X A X^{-1} = X X^{-1} B X X^{-1}$  .....(ii)
  - Since  $X X^{-1} = E$ , one may write eq.(ii) as  $X A X^{-1} = B$ .
  - In other words, the conjugate properties of A and E is mutual.
- **Class :** A complete set of elements, such has A,B,C,.....etc. of a group, which are mutually conjugate with each other, is called a class. each class is completely determined by any one element of it.

- For,example,if an element A of a group is known we obtain the whole class by forming the product  $X A X^{-1}$  where X is successively every element of the group.the unit element E of any group is a class by itself, Since for every other element of the group  $X E X^{-1} = E$ .it is impotent to note that a class of a group is not a subgroup because a class does not have to contain a unit element.

- We can now show that the  $C_{3v}$  group which has six elements  $E, C_3^1, C_3^2, \sigma va, \sigma vb, \sigma vc$  has three classes of operations. The identity operation E is a class by itself. The  $C_3^1$  and  $C_3^2$  form a class can be shown by examining their similarity transformation eq.(i) with every other element of the group

- $E^{-1} C_3^1 E = E C_3^1 E = C_3^1$
- $C_3^{-1} C_3^1 C_3^1 = C_3^2 C_3^1 C_3^1 = C_3^2 C_3^2 = C_3^1$
- $C_3^1 C_3^1 C_3^2 = C_3^1 C_3^3 = C_3^1 E = C_3^1$

- $\sigma va C_3^1 \sigma va = \sigma va \sigma vc = C_3^2$
- $\sigma vb C_3^1 \sigma vb = \sigma vb \sigma va = C_3^2$
- $\sigma vc C_3^1 \sigma vc = \sigma vc \sigma vb = C_3^2$
- similarity, the similarity transformation of  $C_3^2$  by E,  $C_3^1$  and  $C_3^2$  gives  $C_3^2$  and by  $\sigma va$ ,  $\sigma vb$  and  $\sigma vc$  gives  $C_3^1$ . Therefore,  $C_3^2$  and  $C_3^1$  are conjugate and the members of the same class. In an exactly same manner, the three  $\sigma v$ -planes can be shown to belong to a class. The three classes of the  $C_{3v}$  group are as follows,
  - E, ( $C_3^1$  and  $C_3^2$ ), ( $\sigma va, \sigma vb, \sigma vc$ )
  - They can be written as E,  $2C_3$ ,  $3\sigma v$ .

## Reference Books :

1. Introductory Quantum Chemistry-4<sup>th</sup> edition By A.K.Chandra.
2. Group Theory and its chemical applications. By P.K. Bhattacharya.
3. internet and Wikipedia source.

THANK YOU